# Complex Analysis,
# Operator Theory, and Approximation

**Conference dedicated to the memory of Franz Peherstorfer**

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JOHANNES KEPLER UNIVERSITÄT LINZ | JKU

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| | S, 24.07.11 | M, 25.07.11 | **Tu, 26.07.11 Franz' Birthday** | W, 27.07.11 | Th, 28.07.11 | F, 29.07.11 |
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| 09:00 | | Opening | Lagomasino | Gesztesy | Poltoratski | Berg |
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| 12:05 | | Lunch | Lunch | Lunch | Lunch | End |
| 14:00 | | Teschl | Kuijlaars | Excursion and conference dinner | Martinez-Finkelshtein | |
| 15:00 | | Denisov | Khrushchev | | Cantero | |
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| 16:30 | | Golinskii | Bogatyrev | | Kheifets | |
| 17:30 | | Poster session | Schiefermayr<br>Moale | | Lukashov | |
| 19:30 | Welcome reception | | | | | |

All lectures will be held in HF 9901 in Hochschulfondsgebäude.

## 3 Abstracts

# Strong asymptotics of Nuttall-Stahl polynomials

**Alexander I. Aptekarev and Maxim L. Yattselev**

Let

$$f(z) = \sum_{k=0}^{\infty} \frac{c_k}{z^{k+1}} \tag{3.1}$$

be a germ of an analytic function that can be analytically continued along any path in the complex plane deprived of a finite set of points, $f \in \mathcal{A}(\overline{\mathbb{C}} \setminus A), \quad \sharp A < \infty$. J. Nuttall has put forward the important relation between the *maximal domain* of holomorphy of $f$ where the function is single-valued and the *domain of convergence* of the diagonal Padé approximants for $f$ constructed solely based on the series representation (3.1). The Padé approximants, which are rational functions and thus single-valued, approximate a single-valued holomorphic branch of $f$ in the domain of their convergence. At the same time most of their poles tend to the boundary of the domain of convergence and the support of their limiting distribution models the system of cuts that makes the function $f$ single-valued. Nuttall has conjectured (and proved for many important special cases) that this system of cuts has *minimal logarithmic capacity* among all other systems converting the function $f$ to a single-valued branch. Thus the domain of convergence corresponds to the *maximal* (in the sense of *minimal* boundary) domain of holomorphy for the analytic function $f \in \mathcal{A}(\overline{\mathbb{C}} \setminus A)$. The complete proof of Nuttall's conjecture (even in a more general setting where the set $A$ has logarithmic capacity 0) was obtained by H. Stahl. In this work, we derive strong asymptotics for the denominators of the diagonal Padé approximants for this problem in a rather general setting. We assume that $A$ is a finite set of branch points of $f$ which have the *algebro-logarithmic character* and which are placed in a *generic position*. The last restriction means that we exclude from our consideration some degenerated "constellations" of the branch points.

# Applications of complex analysis to questions about the volume of the unit ball in Euclidean space

**Christian Berg and Henrik L. Pedersen**

Many authors have proved results about the volume of the unit ball in $\mathbb{R}^n$,

$$\Omega_n = \frac{\pi^{n/2}}{\Gamma(1+n/2)}, \quad n = 1, 2, \ldots, \tag{3.2}$$

in particular in terms of the sequence $f(n) = \Omega_n^{1/(n \ln n)}, n \geq 2$, which tends to $e^{-1/2}$.

In [1] the authors found an integral representation of the one-parameter family of functions

$$F_a(x) = \frac{\ln \Gamma(x+1)}{x \ln(ax)}, \quad x > 0, a > 0. \tag{3.3}$$

From this it is possbile to deduce that $F_a$ extends to a Pick function if and only if $a \geq 1$. We recall that a Pick function is a holomorphic function of the upper half-plane into itself.

Using this result we can prove that $f(n+2), n \geq 0$ is a Hausdorff moment sequence and in particular decreasing and logarithmic convex.

Alzer found recently the best constants $a^*, b^*$ such that for all $n \geq 2$

$$\exp\left(\frac{a^*}{n(\log n)^2}\right) \leq f(n)/f(n+1) < \exp\left(\frac{b^*}{n(\log n)^2}\right). \tag{3.4}$$

In the proof of this result Alzer considered the function

$$G(x) = \left(1 - \frac{\ln x}{\ln(1+x)}\right) x \ln x, \tag{3.5}$$

and proved that $2/3 < G(x) < 1$ for $x \geq 3$. Qi and Guo observed that this follows because $G$ is strictly increasing on $(0, \infty)$. Furthermore, they conjectured that

$$(-1)^{k-1} G^{(k)}(x) > 0 \text{ for } x > 0, k = 1, 2, \ldots, \tag{3.6}$$

or equivalently that $G'$ is a completely monotonic function. We prove this in [2] by finding an integral representation of the holomorphic extension of $G$ to the cut plane. A main difficulty is caused by the fact that $G'$ is not a Stieltjes function.

The authors have been supported by grant 10-083122 from The Danish Council for Independent Research | Natural Sciences.

## References

[1] C. Berg, H. L. Pedersen, *A one parameter family of Pick functions defined by the Gamma function and related to the volume of the unit ball in n-space*. Proc. A.M.S. **139** No. 6 (2011), 2121–2132.

[2] C. Berg, H. L. Pedersen, *A completely monotonic function used in an inequality of Alzer*. Manuscript.

# Effective methods for optimal stability polynomials

**A. B. Bogatyrev**

The optimization problem for the transition function of explicit multistage Runge-Kutta methods was put forward more than 50 years ago.

Among all degree $n$ real polynomials $P_n(x)$ that approximate the exponential function $\exp(x)$ at $x = 0$ to a given order $p < n$ find the one (which has the name "optimal stability polynomial") that remains within the limits $-1 \leq P_n(x) \leq +1$ on the largest possible interval $[-L, 0]$, $L > 0$ of the real axis.

The direct numerical solution of the above problem is notorious for its difficulty for practically interesting degrees $n > 1000$. We suggest an analytical formula for the solution where a few auxiliary parameters still have to be determined. Say, for $p = 3$ we have just 4 additional unknowns (which have meaning of the moduli of certain algebraic curve) and this number is independent of the degree $n$ of the polynomial.

# $L^p$ integrability of polyharmonic functions

**Alexander Borichev**

Given $p \in (0, 1)$ and $\alpha \in \mathbb{R}$, we study polyharmonic functions $u$ on the unit disc $\mathbb{D}$ such that

$$\int_{\mathbb{D}} |u(z)|^p (1 - |z|^2)^{-\alpha} \, dm_2(z) < \infty.$$

The talk is based on a joint work with Haakan Hedenmalm.

The author was supported by the ANR projects DYNOP and FRAB.

# A special class of orthogonal rational functions on the unit circle, including the associated rational functions

**K. Deckers, <u>M. J. Cantero</u>, L. Moral and L. Velázquez**

A special class of orthogonal rational functions (ORFs) is presented in this talk. Starting with a sequence of ORFs and the corresponding rational functions of the second kind, we define a new sequence as a linear combination of the previous ones, the coefficients of this linear combination being self-reciprocal rational functions. We show that, under very general conditions on the self-reciprocal coefficients, this new sequence satisfies orthogonality conditions as well as a recurrence relation. Further, we identify the Carathéodory function of the corresponding orthogonality measure in terms of such self-reciprocal coefficients.

The new class under study includes the associated rational functions as a particular case. As a consequence of the previous general analysis, we obtain explicit representations for the associated rational functions of arbitrary order, as well as for the related Carathéodory function. Such representations are used to find new properties of the associated rational functions.

# Szegő asymptotics

**Jacob Stordal Christiansen**

The landmark paper [2] of Peherstorfer–Yuditskii shows that one can prove strong asymptotics for orthogonal polynomials of measures supported on sets in $\mathbb{R}$ with infinitely many gaps – so-called homogeneous sets. In the talk, I will discuss a different approach to obtain power asymptotics (aka Szegő asymptotics) that relies on a theorem of Remling [3]. For finite gap sets, this approach was employed in [1] and the aim is to push the method to also cover infinite gap sets of Parreau–Widom type.

If time allows, I will also discuss conditions weaker than the Szegő condition but still sufficient for Szegő asymptotics.

The author was supported by a Steno Research Grant (09-064947) from the Danish Research Council for Nature and Universe.

## References

[1] J. S. Christiansen, B. Simon, and M. Zinchenko, *Finite gap Jacobi matrices, II. The Szegő class*, Constr. Approx. **33** (2011), 365–403.

[2] F. Peherstorfer and P. Yuditskii, *Asymptotic behavior of polynomials orthonormal on a homogeneous set*, J. Anal. Math. **89** (2003), 113–154.

[3] C. Remling, *The absolutely continuous spectrum of Jacobi matrices*, Ann. of Math. **2**, to appear.

# Sharp results for the long-time asymptotics of solutions to the one-dimensional wave equation

**Sergey Denisov**

We consider the Cauchy problem for the wave equation

$$\begin{aligned} y_{tt}(x,t) = y_{xx}(x,t) - q(x)y(x,t), \quad y(x,0) = \phi(x), \quad y_t(x,0) = \psi(x), \\ y(0,t) = 0, \quad t \geq 0, x \in \mathbb{R}^+ \end{aligned} \tag{3.7}$$

where $q(x)$ is time-independent and real-valued. We address the following question in perturbation theory: if $q \in L^p(\mathbb{R}^+)$, what is the largest possible $p$ for which the wave described by (3.7) “propagates” as $t \to \infty$? We prove that this $p = 2$ and establish the long-time asymptotics for $y(x,t)$. The methods are heavily based on the technique developed earlier for the asymptotical analysis of polynomials orthogonal on the unit circle in the polynomial Szegő case.

The author was supported by the NSF Grant DMS-0758239.

# Orthogonal polynomials and the Painlevé equations

**Galina Filipuk**

The talk is based on the paper *Recurrence coefficients of generalized Meixner polynomials and Painlevé equations* [J. Phys. A: Math. Theor. 44 (2011) 035202] with Walter Van Assche and Lies Boelen (KULeuven).

We consider a semi-classical version of the Meixner weight depending on two parameters and the associated set of orthogonal polynomials. These polynomials satisfy a three-term recurrence relation. We show that the coefficients appearing in this relation, when viewed as functions of one of the parameters, satisfy one of Chazy's second-degree Painlevé equations, which can be reduced to the fifth Painlevé equation.

# Approximation in the unit disk based on Radon projections

**I. Georgieva, C. Hofreither, C. Koutschan,**
**V. Pillwein, T. Thanatipanonda and R. Uluchev**

Given information about a function in two variables, consisting of a finite number of values of its Radon projections, i.e., integrals along some chords of the unit circle and function values, we study the problem of interpolating these data by a bivariate polynomial. In the special case when the function is harmonic we consider interpolation by harmonic polynomials. With the help of symbolic summation techniques we show that this interpolation problem has a unique solution in the case when the chords form a regular polygon. Numerical experiments for this and more general cases are presented.

I. Georgieva and R. Uluchev were supported by Bulgarian National Science Fund under Grant No. DDVU 02/30, 2010.

# Bivariate orthogonal polynomials and the factorization of bivariate positive trigonometric polynomials

**Jeff Geronimo**

Given a positive multivariate polynomial an important and difficult problem is to make the positivity manifest. Sometimes this can be accomplished by writing the polynomial as a sum of squares or square magnitudes of other polynomials. I will discuss how the theory of bivariate orthogonal polynomials provides methods for writing classes of bivariate positive trigonometric polynomials as a magnitude square of another polynomial. This is work with Plamen Iliev.

# On a generalization of the spectral problem underlying the Camassa–Holm hierarchy

**Fritz Gesztesy** and **Rudi Weikard**

Using a Birman–Schwinger-type operator approach, we study generalizations of the left-definite spectral problem underlying the Camassa–Holm hierarchy. We re-derive a number of known results, including basic Floquet theoretic facts, considerably relaxing the conditions on the coefficients (permitting certain measures and distributions).

We also illustrate the manner in which the Birman–Schwinger-type approach is most natural in the sense that it yields necessary and sufficient conditions for certain relative boundedness and relative compactness results.

# Blaschke-type conditions for a class of analytic functions in the unit disk and applications

**Leonid Golinskii**

The main object under consideration is the class $\mathcal{H}(\mathcal{E},\rho)$ of analytic functions in the unit disk of finite order at most $\rho > 0$ having an arbitrary closed set $E$ on the unit circle as the set of singular points

$$\log|f(z)| \le \frac{C(f)}{d^{\rho}(z,E)}\,,$$

$d(z,E)$ is the distance from $z$ to $E$. We prove the Blaschke-type condition for zeros of such functions. The similar result is obtained for related subharmonic functions in the disk and their Riesz measures. The inverse problem of reconstructing a function $f \in \mathcal{H}(\mathcal{E},\rho)$ from its zero set is studied. The applications to the operator theory (the discrete spectrum of the Schatten class perturbations of normal operators) and to the function theory (the critical points of Blaschke products) are considered.

# Peherstorfer-Yuditskii scattering approach to Ryckman's class of Jacobi matrices

**Alexander Kheifets**

Analyzing (rather complicated) Geronimus relations, E. Ryckman [2006] showed that there was a natural correspondence between Strong Szegő class of Schur parameters ($\sum_{n=1}^{\infty} n|\alpha_n|^2 < \infty$) on the one hand and the following class of Jacobi matrices on the other

$$\sum_{n=0}^{\infty} n \left| \sum_{k=n}^{\infty} (p_k - 1) \right|^2 < \infty \quad \text{and} \quad \sum_{n=0}^{\infty} n \left| \sum_{k=n}^{\infty} q_k \right|^2 < \infty,$$

where $q_n$ and $p_n$ are the diagonal and the off-diagonal entries, respectively. Applying B. Golinskii – I. Ibragimov Theorem, E. Ryckman characterized the spectral measures of this class of Jacobi matrices. We use the scattering approach (developed by F. Pehersorfer and P. Yuditskii [2001, 2003] and A. Volberg and P. Yuditskii [2001]) to analyze the Ryckman's result. This approach, was, in particular, applied to the Strong Szegő class by L. Golinskki, A. Kheifets, F. Peherstorfer and P. Yuditskii [2011].

# Periodic measures by Franz Peherstorfer

**Sergey Khrushchev**

The talk is a survey of Peherstorfer's contribution to the theory of measures with periodic Verblunsky parameters from the point of view of the theory of Wall Pairs.

# Franz Peherstorfer's work on extremal problems in Approximation Theory

**András Kroó**

In this talk we shall attempt to give a survey of 30 years of Franz Peherstorfer's brilliant work on extremal problems in Approximation Theory. Extending and enhancing ideas of Chebyshev, Zolotarev, Bernstein and Achieser, Franz obtained deep results on representation of Zolotarev polynomials in $L_1$ and $L_\infty$ norms, asymptotic representation of weighted $L_p$ minimal polynomials, interlacing and distribution of zeros of various extremal polynomials.

# Orthogonal polynomials in the normal matrix model

**Pavel Bleher and Arno Kuijlaars**

The normal matrix model is a random matrix model defined on complex matrices. The eigenvalues in this model fill a two-dimensional region in the complex plane as the size of the matrices tends to infinity. Orthogonal polynomials with respect to a planar measure are a main tool in the analysis.

In many interesting cases, however, the orthogonality is not well-defined, since the integrals that define the orthogonality are divergent. We will present a way to redefine the orthogonality in terms of a well-defined Hermitian form. This reformulation allows for a Riemann-Hilbert characterization. For the special case of a cubic potential it is possible to do a complete steepest descent analysis on the Riemann-Hilbert problem, which leads to strong asymptotics of the orthogonal polynomials, and in particular to the two-dimensional domain where the eigenvalues accumulate.

# An interpolatory estimate for the UMD–valued directional Haar projection

**Richard Lechner**

The Calculus of Variations, in particular the theory of compensated compactness has long been a source of hard problems in harmonic analysis. One development started with the work of F. Murat and L. Tartar, where the decisive theorems were on Fourier multipliers of Hörmander type. Using time–frequency localizations relying on Littlewood–Paley and wavelet expansions, as well as modern Calderon–Zygmund theory S. Müller and J. Lee, P. F. X. Müller, S. Müller extended and strengthened the results obtained by Fourier multiplier methods.

In this paper we avoid this time–frequency localization by utilizing T. Figiel's canonical martingale decomposition instead, which permits us to further extend the results to the Bochner–Lebesgue space $L^p_X(\mathbb{R}^n)$, provided $X$ satisfies the UMD–property. Given $n \geq 2$, $1 < p < \infty$ and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n) \in \{0,1\}^n \setminus \{0\}$, define the directional Haar projection by $P^{(\varepsilon)}u = \sum_{Q \in \mathcal{Q}} \langle u, h_Q^{(\varepsilon)} \rangle \, h_Q^{(\varepsilon)} \, |Q|^{-1}$, for all $u \in L^p_X(\mathbb{R}^n)$. The Haar function $h_Q^{(\varepsilon)}$ is supported on the dyadic cube $Q$, has mean zero in each of the coordinates $i$ whenever $\varepsilon_i = 1$, and $h_Q^{(\varepsilon)}(Q) = \{\pm 1\}$. Let $R_i$, $1 \leq i \leq n$ denote the Riesz-transform acting on the $i$–th coordinate. If $\varepsilon_{i_0} = 1$ and $L^p_X$ has type $\mathcal{T}(L^p_X)$, the main result of this paper is the interpolatory estimate

$$\|P^{(\varepsilon)}u\|_{L^p_X} \leq C \, \|u\|_{L^p_X}^{1/\mathcal{T}(L^p_X)} \cdot \|R_{i_0}u\|_{L^p_X}^{1-1/\mathcal{T}(L^p_X)},$$

holding true for all $u \in L^p_X$, where $C > 0$ depends only on $n$, $p$, $X$ and $\mathcal{T}(L^p_X)$.

# On the perfectness of Nikishin systems

G. López Lagomasino and U. Fidalgo Prieto

Recently we proved that Nikishin systems are perfect assuming that the measures that generate the system were supported on bounded non-intersecting intervals of the real line. Here, we study the case when the generating measures are allowed to have unbounded support with common end points. This result allows to obtain an extension of the Stieltjes theorem to type II Hermite-Padé approximation of Nikishin systems.

# A class of polynomial interpolation matrices with optimal order of the Lebesgue constants

Alexey Lukashov

**Definition 3.1.** A sequence of polynomials $\{w_n(x)\}$ is called flat on a compact $K \subset \mathbb{R}$, if there are constants $C_1, C_2 > 0$ such that $C_1 < w_n(x) < C_2$ for all $x \in K, n \in \mathbb{N}$.

A series of results will be presented about estimation of the Lebesgue constants for polynomial interpolation at the zeros or at the extremas of the Chebyshev polynomials with weights $1/w_n(x)$ on $K$, where $\{w_n(x)\}$ is a flat sequence and $K$ consists of one or several intervals. In particular, under additional restrictions these constants have optimal order $O(\log n)$. Moreover, the case of interpolation at inner extremas only is considered as well. Note that for the last case an estimate for the Lebesgue constants was given in [1, Lemma 4.5].

## References

[1] A. Kroo, F. Peherstorfer, *Asymptotic representation of weighted $L_\infty$- and $L_1$-minimal polynomials.* Math. Proc. Cambridge Philos. Soc. **144** (2008), 241-254.

# Generators of the group of rational spectral transformations for non-trivial C-functions

Francisco Marcellán and Kenier Castillo

In this paper we consider perturbations of sequences of orthogonal polynomials associated with a Hermitian linear functional $\mathcal{L}$ using spectral transformations of the corresponding $\mathcal{C}$-function $F_{\mathcal{L}}$.

A rational spectral transformation is defined by the following rational expression

$$F_{\widetilde{\mathcal{L}}}(z) = \left(\frac{AF_{\mathcal{L}} + B}{CF_{\mathcal{L}} + D}\right)(z)$$

where $A$, $B$, $C$, and $D$ are coprime Laurent polynomials. Examples of rational spectral transformations are studied in [1].

We show that a rational spectral transformation of $F_{\mathcal{L}}$ with Laurent polynomial coefficients is a finite composition of four fundamental elementary spectral transformations.

The counterpart of our result for orthogonal polynomials associated with nontrivial probability measures supported on the real line has been done in [2].

The authors were supported by the Ministerio de Ciencia e Innovación of Spain, project number MTM2009-12740-C03-01.

## References

[1] F. Peherstorfer, *A special class of polynomials orthogonal on the unit circle including the associated polynomials*, Constr. Approx. **12** (1996), 161-185.

[2] A. Zhedanov, *Rational spectral transformations and orthogonal polynomials*, J. Comput. Appl. Math. **85** (1997), 67-86.

# Asymptotics of the $L^2$ norms of derivatives of OPUC

**Andrei Martínez–Finkelshtein and Barry Simon**

Let $\mu$ be a non-trivial Borel measure on the unit circle $\mathbb{T}$, and $\{\varphi_n\}$ the corresponding sequence of orthonormal polynomials (OPUC). Let us denote by

$$\|f\|_{L^2_\mu} = \left(\int |f|^2 \, d\mu\right)^{1/2}$$

the weighted $L^2$ norm. In this talk we will focus on one question from the asymptotic theory of OPUC. Namely, we say that $\mu$ has normal $L^2$–derivative behavior (is *normal*, for short), if

$$\lim_n \left\| \frac{\varphi_n'}{n} \right\|_{L^2_\mu} = 1 \,. \tag{3.8}$$

We will discuss some connections of this notion with different relevant objects in the theory of OPUC, and present some necessary/sufficient conditions for (3.8).

# An explicit class of min-max polynomials on the ball and on the sphere

**Ionela Moale** and **Franz Peherstorfer**

Let $\Pi^d_{n+m-1}$ denote the set of polynomials in $d$ variables of total degree less than or equal to $n+m-1$ with real coefficients and let $\mathcal{P}(x)$, $x=(x_1,...,x_d)$, be a given homogeneous polynomial of degree $n+m$ in $d$ variables with real coefficients. We look for a polynomial $p^* \in \Pi^d_{n+m-1}$, such that $\mathcal{P}-p^*$ has the least max norm on the unit ball and the unit sphere in dimension $d \geq 3$ and call $\mathcal{P}-p^*$ a min-max polynomial. For every $n, m \in \mathbb{N}$, we derive min-max polynomials for $\mathcal{P}$ of the form $\mathcal{P}(x) = P_n(x')x_d^m$, with $x' = (x_1, ..., x_{d-1})$, where $P_n(x')$ is the product of real homogeneous harmonic polynomials in two variables. In particular, for every $m \in \mathbb{N}$ min-max polynomials for the monomials $x_1...x_{d-1}x_d^m$ are obtained.

The author was supported by the Austrian Science Fund FWF, project number: P20413-N18.

# The Lebesgue constant for the periodic Franklin system

**Markus Passenbrunner**

We identify the torus with the unit interval $[0,1)$ and let $n, \nu \in \mathbb{N}$, $1 \leq \nu \leq n-1$ and $N := n+\nu$. Then we define the (partially equally spaced) knots

$$t_j = \begin{cases} \frac{j}{2n}, & \text{for } j = 0, \dots, 2\nu, \\ \frac{j-\nu}{n}, & \text{for } j = 2\nu+1, \dots, N-1. \end{cases}$$

Furthermore, given $n, \nu$ we let $V_{n,\nu}$ be the space of piecewise linear continuous functions on the torus with knots $\{t_j : 0 \leq j \leq N-1\}$. Finally, let $P_{n,\nu}$ be the orthogonal projection operator of $L^2([0,1))$ onto $V_{n,\nu}$. We obtain the following exact bound of the $L^\infty$ norm of these projection operators:

$$\lim_{n\to\infty, \nu=1} \|P_{n,\nu} : L^\infty \to L^\infty\| = \sup_{n\in\mathbb{N}, 0\leq\nu\leq n} \|P_{n,\nu} : L^\infty \to L^\infty\| = 2 + \frac{33-18\sqrt{3}}{13}.$$

This shows in particular that the Lebesgue constant of the classical Franklin orthonormal system on the torus is $2 + \frac{33-18\sqrt{3}}{13}$.

The author was supported by the Austrian Science Fund FWF, project number: P20166-N18.

# Orthogonal Polynomials, Finite Band Jacobi Matrices, and Random Matrices

**Leonid Pastur**

We discuss recent results on asymptotics of orthogonal polynomials stressing their spectral aspects and similarity in two cases considered. They concern the polynomials orthonormal on a finite union of disjoint intervals with respect to the Szegö weight and polynomials orthonormal on the whole axis with respect to varying weights and having the same union of intervals as the set of oscillations of asymptotics. In both cases we construct double infinite finite band Jacobi matrices with generically quasiperiodic coefficients and finite band spectrum and show that they are related via an isospectral deformation. Related results on asymptotic eigenvalue distribution of a class of random matrices of large size are also briefly discussed.

# Elementary solutions of the Bernstein problem on two intervals

**Florian Pausinger**

First we note that the best polynomial approximation to $|x|$ on the set, which consists of an interval on the positive half axis and a point on the negative half axis, can be given by means of the classical Chebyshev polynomials. Then we explore the cases when a solution of the related problem on two intervals can be given in elementary functions.

This work is supported by the Austrian Science Fund (FWF), Project P22025-N18.

# Completeness of complex exponentials in $L^2$-spaces: Gap and Type problems

**Alexei Poltoratski**

One of the basic problems of Harmonic analysis is to determine if a given collection of functions is complete in a given Hilbert space. A classical theorem by Beurling and Malliavin solved such a problem in the case when the space is $L^2$ on an interval and the collection consists of complex exponentials. Two closely related problems, the so-called Gap and Type Problems, posted over 50 years ago, remained open until recently. In my talk I will present solutions to the Gap and Type problems and discuss their connections with adjacent fields of analysis.

# Inequalities for norm of Chebyshev polynomials and a refinement of the Bernstein-Walsh lemma

**Klaus Schiefermayr**

Given a compact set in the complex plane, we consider the behavior of minimal norms

$$L_n(S) := \min_{P \in \mathbb{P}_n} \|P\|_S,$$

where $\|\cdot\|_S$ denotes the supremum norm on $S$ and $\mathbb{P}_n$ is one of the following sets:

1. $\mathbb{P}_n$ is the set of all monic polynomials of degree $n$.
2. $\mathbb{P}_n$ is the set of all polynomials $P$ of degree $\leq n$ with the additional constraint $P(0) = 1$.

In both cases, the limit

$$c(S) := \lim_{n \to \infty} \sqrt[n]{L_n(S)}$$

exists and is called in the first case Chebyshev constant (or logarithmic capacity), in the second case the estimated asymptotic convergence factor (this notation comes from an important application in numerical linear algebra).

For both cases, we present some inequalities for $L_n(S)$ if $S$ is real. From these inequalities, we were able to improve the Bernstein-Walsh Lemma for real sets.

# Natural boundaries and spectral theory

**Barry Simon**

This talk describes joint work with Jonathan Breuer. The last ten years has seen considerable understanding of the spectrum of general Jacobi matrices in terms of its right limits due to work of Last-Simon and especially Remling. We have discovered that analogs of these ideas can be used to understand when a power series (with bounded Taylor coefficients) has a natural boundary on the unit circle. One recovers and (within the class of bounded coefficients) improves many classical results. The main theorem depends on little more than the notions of right limit and reflectionless double power series (that we carry over from the theory of Jacobi matrices) and a clever lemma proven by M. Riesz in 1916 (using the maximum principle). This will be a colloquium-level talk that should be accessible to anyone with a good complex variables course.

# Error bounds for Gaussian quadrature formulae with Bernstein-Szegő weight functions

**Aleksandar V. Pejčev and Miodrag M. Spalević**

We study the kernels $K_n(z) = K_n(z, w)$ in the remainder terms $R_n(f)$ of the Gaussian quadrature formulas

$$\int_{-1}^{1} f(t)w(t)\,dt = G_n[f] + R_n(f), \quad G_n[f] = \sum_{\nu=1}^{n} \lambda_\nu f(\tau_\nu) \quad (n \in \mathbb{N})$$

for analytic functions inside elliptical contours $\mathcal{E}_\rho$ with foci at $\mp 1$ and the sum of semi-axes $\rho > 1$, where $w$ is a nonnegative and integrable weight function of Bernstein-Szegő type. The derivation of effective bounds for $|R_n(f)|$ is possible if good estimates for $\max_{z\in\mathcal{E}_\rho} |K_n(z)|$ are available, especially if we know the location of the extremal point $\eta \in \mathcal{E}_\rho$ at which $|K_n|$ attains its maximum. In such a case, instead of looking for upper bounds for $\max_{z\in\mathcal{E}_\rho} |K_n(z)|$ one can simply try to calculate $|K_n(\eta, w)|$. In the case under consideration the location on the elliptic contours where the modulus of the kernel attains its maximum value is investigated. This leads to effective bounds for $|R_n(f)|$.

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# Long-time asymptotics of the Toda lattice for asymptotically periodic initial conditions

**Gerald Teschl and Spyros Kamvissis**

We consider the stability of the periodic Toda lattice under a short range perturbation. We prove that the perturbed lattice asymptotically approaches a modulated lattice.

More precisely, let $g$ be the genus of the hyperelliptic curve associated with the unperturbed solution. We show that, apart from the phenomenon of the solitons travelling on the quasi-periodic background, the $n/t$-pane contains $g + 2$ areas where the perturbed solution is close to a finite-gap solution in the same isospectral torus. In between there are $g + 1$ regions where the perturbed solution is asymptotically close to a modulated lattice which undergoes a continuous phase transition (in the Jacobian variety) and which interpolates between these isospectral solutions. In the special case of the free lattice ($g = 0$) the isospectral torus consists of just one point and we recover the classical result.

Both the solutions in the isospectral torus and the phase transition are explicitly characterized in terms of Abelian integrals on the underlying hyperelliptic curve.

Our method relies on the equivalence of the inverse spectral problem to a matrix Riemann–Hilbert problem defined on the hyperelliptic curve and generalizes the so-called nonlinear stationary-phase-steepest-descent method for Riemann–Hilbert problem deformations to Riemann surfaces.

The author was supported by the Austrian Science Fund FWF, project number: Y330.

# The norm of Chebyshev polynomials

**Vilmos Totik**

Franz Peherstorfer and his students made significant contributions to understanding Chebyshev polynomials $T_n$ on non-connected compact sets $K$. This talk will use some of their works for estimating the smallest possible norm. The main topic is how close $\|T_n\|_K$ can get to the theoretical lower limit $\mathrm{cap}(K)^n$ ($2\mathrm{cap}(K)^n$ when $K \subset \mathbf{R}$). It will be shown that some sequences of $n$ behave better than others, and simultaneous Diophantine approximation of harmonic measures determine the rate of $\|T_n\|_K/\mathrm{cap}(K)^n - 1$. To get the correct (exponentially small) error term when $K$ consists of a finite number of analytic curves, a substitute of Faber polynomials is introduced for a finite union of Jordan domains. The results are closely related to the question how well $K$ can be approximated in Hausdorff distance by lemniscates.

# On trigonometric polynomials of half-integer order deviated least from zero on an interval

**Alexey Lukashov and Sergey Tyshkevich**

Let $\mathcal{T}_N^{(A,B)}$ be the class of trigonometric polynomials of the form

$$\tau_N(\varphi) = A\cos\frac{N}{2}\varphi + B\sin\frac{N}{2}\varphi + c_1\cos\left(\frac{N}{2}-1\right)\varphi + d_1\sin\left(\frac{N}{2}-1\right)\varphi + \dots$$
$$\dots + c_{\left[\frac{N}{2}\right]}\cos\left(\frac{N}{2}-\left[\frac{N}{2}\right]\right)\varphi + d_{\left[\frac{N}{2}\right]}\sin\left(\frac{N}{2}-\left[\frac{N}{2}\right]\right)\varphi,$$
$$A, B, c_1, d_1, \dots, c_{\left[\frac{N}{2}\right]}, d_{\left[\frac{N}{2}\right]} \in \mathbb{R}.$$

The solution $\tau_N^*(\varphi)$ of the problem

$$\max_{\varphi\in\mathcal{E}}|\tau_N^*(\varphi)| = \min_{\tau_N\in\mathcal{T}_N^{(A,B)}}\max_{\varphi\in\mathcal{E}}|\tau_N(\varphi)|$$

is given depending on $\mathcal{E} = [\alpha_1, \alpha_2],\ 0 < \alpha_2 - \alpha_1 < 2\pi$, and $A, B:\ A^2 + B^2 = 1$.

In fact three possible types of solution may appear:

- $\tau_N^*(\varphi) = \cos\left(\frac{N}{2}\varphi + \gamma\right)$,
- transformed Videnskii's type polynomials [1],
- "elliptic trigonometric polynomials" from [2, Corollary 4].

## References


[1] V. S. Videnskii, *Extremalnye otsenki proisvodnykh polynoma na otreske// Issledovanija po sovremennym problemam konstruktivnoj teorii functsij*, Sb. statej pod red. V. I. Smirnova.-M.: Fismatlit (1961), 98-106 (in russian).

[2] A. L. Lukashov, F. Peherstorfer, *Zeros of polynomials orthogonal on two arcs of the unit circle*, J. Approx. Theory. **132** (2005), 42–71.


# Sharp estimates of singular integrals


**Alexander Volberg**

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